# Laplace Equations

$$\boxed{\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt}$$

## Basic Defns

$\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$

$\mathcal{L}\{\sin kt\} = \dfrac{k}{s^2+k^2}$

$\mathcal{L}\{t^n e^{at}\} = \dfrac{n!}{(s-a)^{n+1}}$ (n is positive only)

$\mathcal{L}\{\cos kt\} = \dfrac{s}{s^2+k^2}$

$\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$

$\mathcal{L}\{\sinh kt\} = \dfrac{k}{s^2-k^2}$

$\mathcal{L}\{1\} = \dfrac{1}{s}$

$\mathcal{L}\{\cosh kt\} = \dfrac{s}{s^2-k^2}$

$\mathcal{L}\{t\} = \dfrac{1}{s^2}$

## Derivatives

$\mathcal{L}\{f'(t)\} = sF(s) - f(0) = s\mathcal{L}\{f\} - f(0)$

$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0) = s^2\mathcal{L}\{f\} - sf(0) - f'(0)$

$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

$\mathcal{L}\{f^n(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots f^{(n-1)}(0)$

## First Translation Theorem

$\mathcal{L}\{e^{at} f(t)\} = F(s-a) = \mathcal{L}\{f(t)\}\big|_{s \to s-a}$

$e^{at} \downarrow$

## Second Translation Theorem

$\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}F(s)$

where $\mathcal{U}(t-a) = \begin{cases} 0 & 0 \le t < a \\ 1 & t \ge a \end{cases}$ $\qquad \mathcal{L}\{\mathcal{U}(t-a)\} = \dfrac{e^{-as}}{s}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n} F(s)$

$\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

$f * g = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$

$\mathcal{L}\{f*g\} = \mathcal{L}\{f\}\mathcal{L}\{g\} = F(s)G(s)$

$\int_0^t f(\tau)\,d\tau = \mathcal{L}^{-1}\left\{\dfrac{F(s)}{s}\right\}$

$\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$